Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures   ☐☐ ☐☐☐ ☐☐☐

In words   _____

_____

Your name   _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	13	13	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free **35% (51 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

A sequence represents the value ($) in a child's piggy bank at the beginning of each week after n weeks of saving. It is defined as $T_{n+1} = T_n + a$ with $T_1 = b$, so that $T_4 = 40$ and $T_5 = 43$.

(a) Determine the value of the constant a and the value of the constant b. (2 marks)

The sequence can also be written in the form $T_n = an + k$.

(b) Determine the value of the constant k. (2 mark)

(c) After how many weeks of saving will the child have $250 in their piggy bank? (2 marks)

Question 2 **(6 marks)**

(a)   A connected planar graph has 14 vertices and 9 faces. Determine the number of edges
this graph has. (2 marks)

(b)   The vertices in the following graph can be split into two distinct groups to demonstrate that
the graph is bipartite. List the vertices in each group. (2 marks)
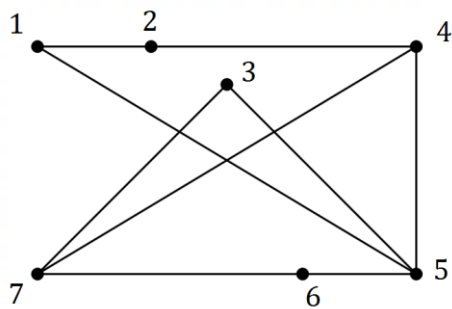
(c)   Determine the number of edges that must be removed from a complete graph with 5
vertices so that it becomes a tree with 5 vertices. (2 marks)

Question 3   (7 marks)

A company runs sightseeing boat trips from several terminals throughout a harbour city. In the graph below, the numbered vertices represent terminals, and the edges represent trips either between the terminals or that start and finish at the same terminal.
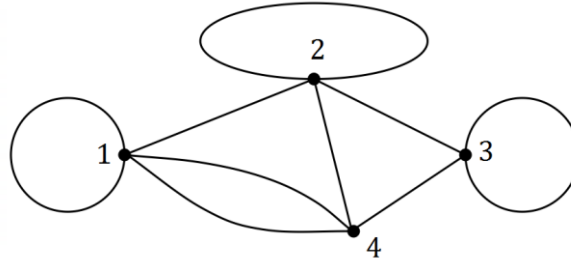
(a)   State two reasons why the graph above is not simple.   (2 marks)

(b)   Complete the one stage adjacency matrix below from the graph.   (2 marks)

$$\begin{array}{c}\\1\\2\\3\\4\end{array}\begin{array}{cccc}1 & 2 & 3 & 4\end{array}\\\left[\begin{array}{cccc} & & & 2\\1 & & 1 & 1\\0 & & & \\2 & & & \end{array}\right]$$

(c)   Describe two conditions necessary for the existence of a semi-Eulerian trail in a graph and state the length of such a trail in the above graph.   (3 marks)

Question 4 (9 marks)

An analyst collected data from a sample of ten trees in a plantation, as shown in the table below.

The variables x and y are the diameter of a tree in centimetres and the daily water use of the tree in litres respectively.

$\hat{y}$ is the predicted daily water use in litres, calculated using the least-squares line $\hat{y} = 2.2x + 0.6$, and R is the residual.

x	12	15	20	22	24	27	31	35	39	43
y	32.5	35	41.5	48.5	52.5	56.5	66	77	90	98.5
$\hat{y}$	27	33.6	-	49	53.4	60	68.8	77.6	86.4	95.2
R	5.5	1.4	-	−0.5	−0.9	−3.5	−2.8	−0.6	-	3.3

Nine of the data points (x, y) are shown on this scatterplot:
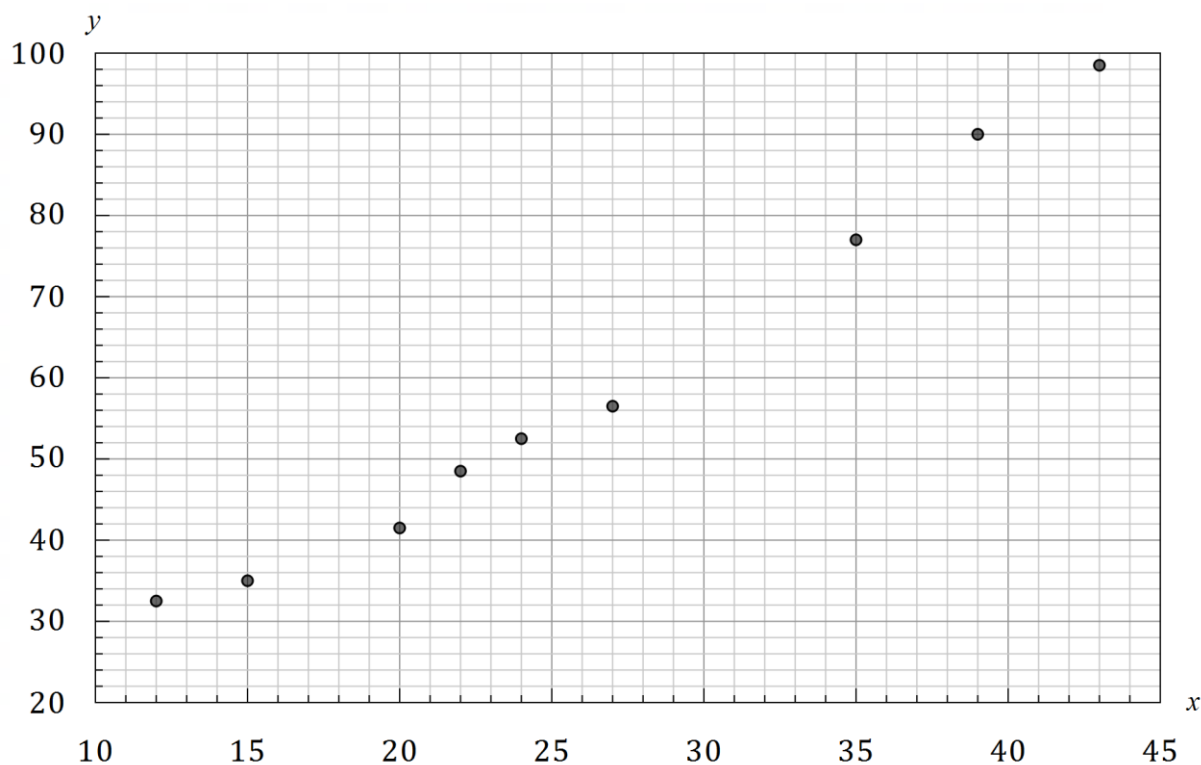
(a)   Add the missing point to the scatterplot. (1 mark)

(b)   Explain how information from the table can be used to draw the least-squares line on the scatterplot and hence draw this line. (2 marks)

See next page

(c)   Determine the value of the residual R when:

   (i) $x = 39$.   (1 mark)

   (ii) $x = 20$.   (2 marks)

(d)   Complete the residual plot on the axes below by adding the two missing residuals.
                                                                                                              (1 marks)
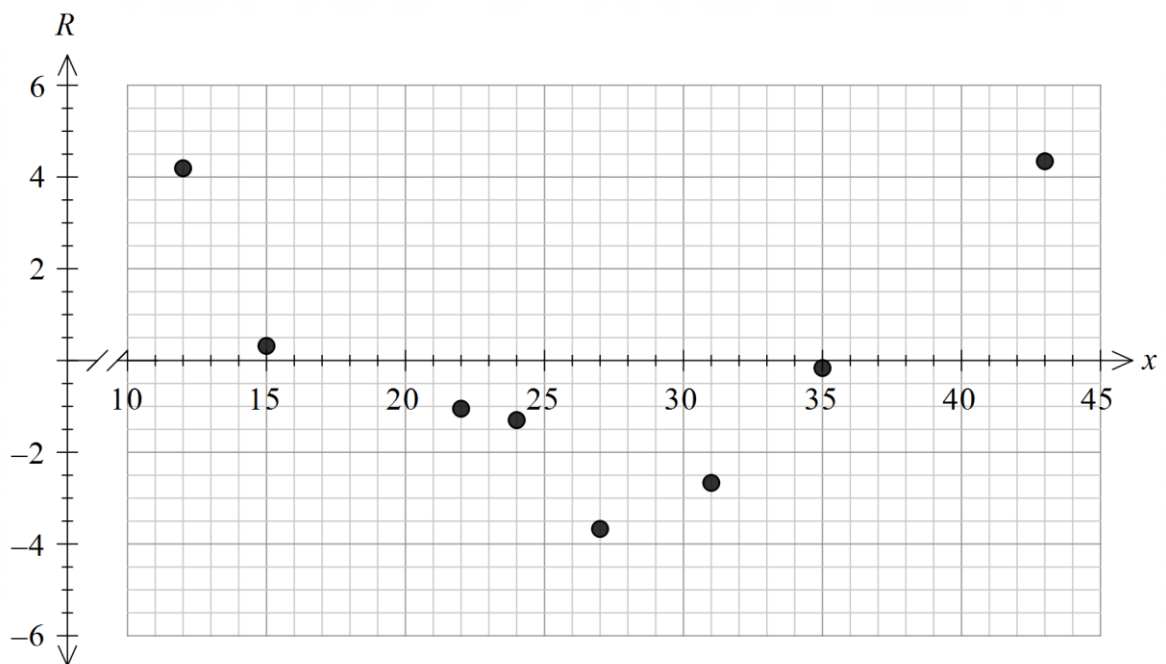
(e)   Comment on the appropriateness of fitting a linear model to the data. Justify your answer.
                                                                                                              (2 marks)

Question 5   **(5 marks)**

A relay team consists of four cadets who must each be assigned to one of the four sections of an assault course in order to minimise their overall time.

The table shows the least time, in minutes, that each cadet has previously taken to complete the different sections.

	Section			
	1	2	3	4
Drew	33	26	32	34
Eve	27	26	29	28
Faye	30	27	30	32
Gem	31	30	27	30

(a) Show use of the Hungarian algorithm to determine the optimum assignment of cadets, writing the assignment in the table below.   (4 marks)

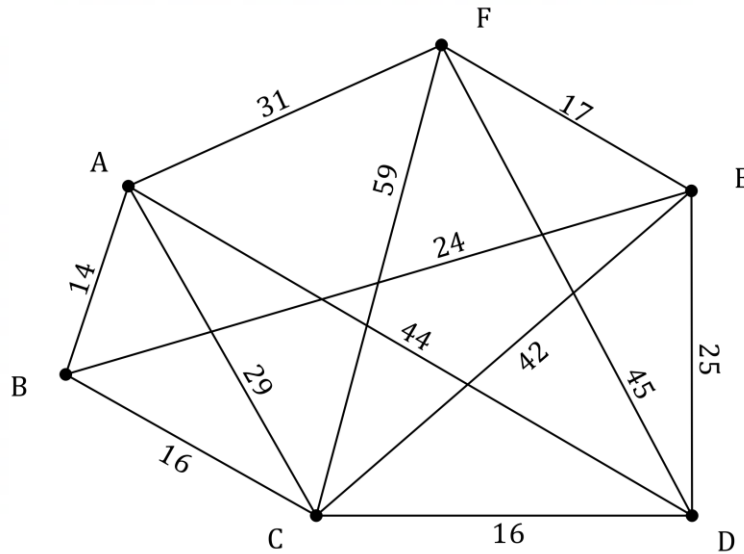| Section	1	2	3	4
Cadet				

(b) State the predicted least overall time for the team to complete the relay.   (1 mark)

Question 6 **(6 marks)**

The edge weights on the graph below represent the time, in milliseconds, to send a data packet between routers on a computer network, represented by the vertices.

(a) Determine the minimum time to send a data packet from router C to router F and state, in order, the routers on this path. (2 marks)

(b) Explain, with use of an example, why the graph in this question is Hamiltonian cycle. (2 marks)

(c) State, with reasoning, the least number of edges that must be removed from the graph so that it is no longer Hamiltonian cycle. (2 marks)
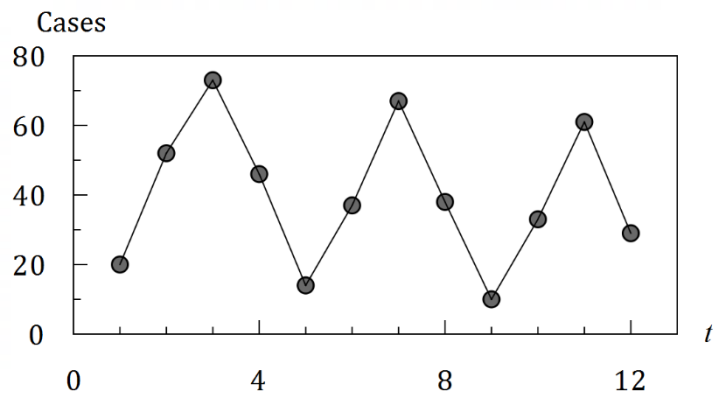
Question 7   **(6 marks)**

A clinic recorded the number of cases of influenza that presented each quarter, and an extract from the data is shown in the table and graph below.

	Total Cases			
	Quarter			
	1	2	3	4
2016	20	52	73	46
2017	14	37	67	38
2018	10	33	61	29

(Year label on left spanning the year rows)

(a)   Describe the trend and seasonality of the data.   (2 marks)

(b)   Calculate the 4-point centred moving average for the number of cases that presented in the third quarter of 2016.   (2 marks)

The 3-point moving average for the number of cases that presented in the fourth quarter of 2018 is 34.

(c)   Determine the number of cases that presented in the first quarter of 2019.   (2 marks)

See next page

Question 8 **(6 marks)**

The network below represents the durations and interdependencies of the 11 activities required to complete a construction project in days. For example, activity M has a duration of 9 days and cannot commence until activities F and L are complete.
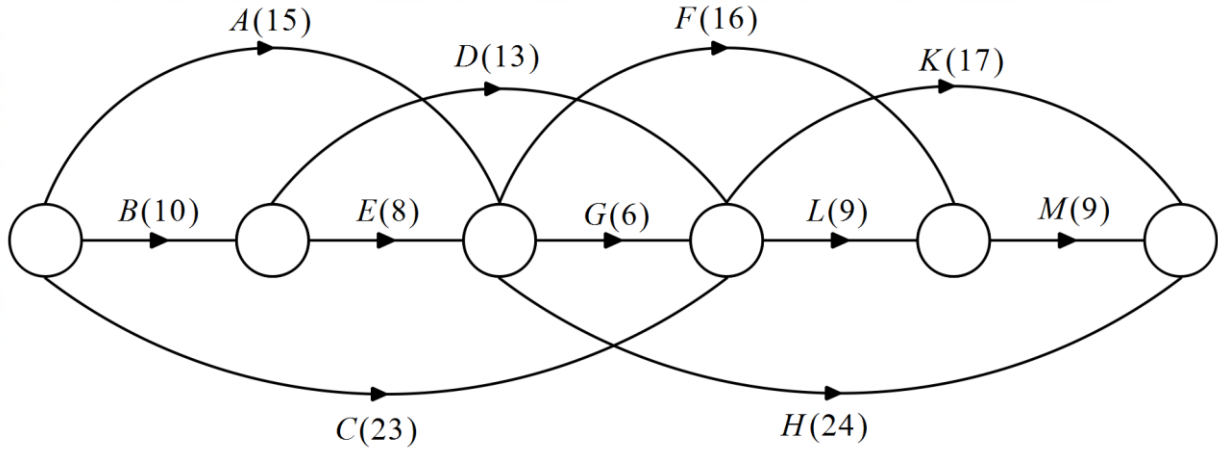
(a) Determine the minimum completion time for the construction project. (2 marks)

(b) Determine which of the non-critical activities has the greatest float time and state the earliest start time and latest start time for this activity. (2 marks)

(c) Proposed changes to the construction project will halve the duration of activity E. Determine the impact this will have on the critical path of the project and its minimum completion time. (2 marks)

End of questions

Supplementary page

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Supplementary page

Question number: _____

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